Bit Allocation for Wavelet Image Compression and Uniform Bit Loss

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Abstract

We explore the distribution of bits among wavelet image subbands that are transmitted across a channel with uniform bit loss, which models interleaved data sent across a packet erasure channel. We develop a closed-form expression relating the source rate-distortion curves to the rate-distortion curves that include the effect of expected bit losses. We use that expression to determine optimal bit assignments for each subband. Our results show that the bit allocation is biased against long codeword indexes; as a result, the bit allocation is more uniform among subbands in the presence of loss than under conditions of no loss. To demonstrate our methods, we use a 3-level discrete wavelet transform and encode each subband with its own balanced tree-structured vector quantizer of block size 2×2.

1 Approach

The wavelet transform is a linear orthonormal transform that concentrates signal energy in a small number of subbands, and any error introduced in the transform domain is quantitively identical in the signal domain. A tree-structured vector quantizer is progressive: a crude representation is transmitted first, followed by better and better approximations in subsequent passes. Our approach to handling the loss of a bit in a codeword is to truncate it to the previously-received bits, thus treating those bits as an index to an internal node in the tree.

2 Adjusted Distortion

We derive an equation for the expected distortion in the presence of bit loss, as seen by the receiver.

- Let r be the rate in bits per vector.

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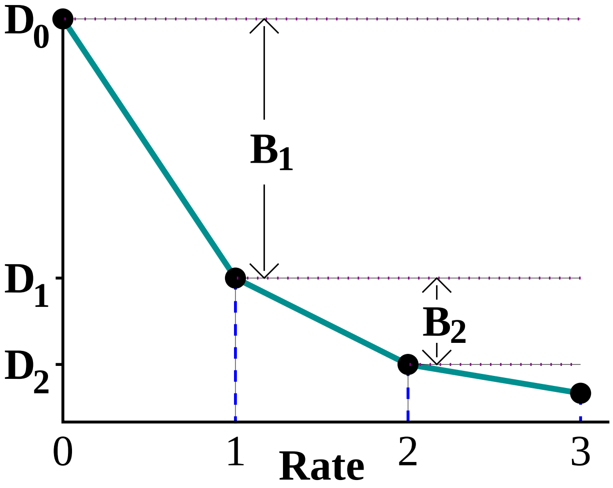

Figure 1: The "benefit" of a bit is the decrease in distortion due to receiving that bit. It is a positive number.

- Let D_r be the distortion at rate r.
- Let P_L be the probability of losing a bit.

We then let

$$P_{U,k} = (1 - P_L)P_{U,k-1} \tag{1}$$

be the probability that bit k is "useful," which we define as the case in which that bit is received and the previous bit is also "useful."

Solving that recurrence yields:

$$P_{U,k} = (1 - P_L)^k. \tag{2}$$

The "benefit" of a bit is the decrease in distortion that results from receiving that bit, as illustrated in Figure 1:

$$B_k = (D_{k-1} - D_k). \tag{3}$$

In the presence of bit loss, the k-th bit no longer reduces the distortion by B_k, but rather by the smaller quantity $P_{U,k}B_k$. That change results in the following equation for the adjusted distortion:

$$D'_r = D_0 - \sum_{k=1}^{r} P_{U,k}B_k. \tag{4}$$

In other words, the adjusted distortion for a given rate is the distortion at rate 0 subtracted by the adjusted benefit of each additional bit.

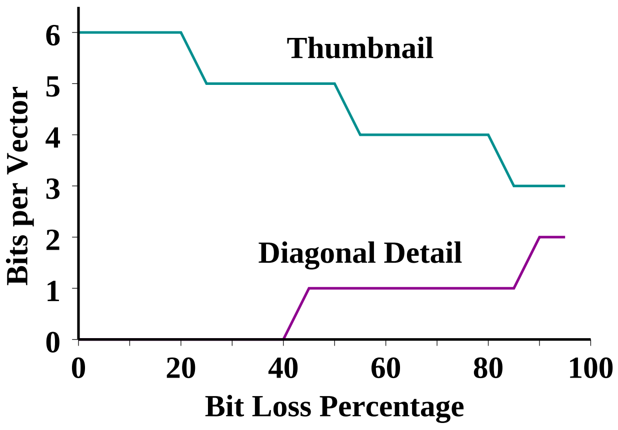

Figure 2: As bit losses increase, fewer bits are assigned to the low-frequency thumbnail subband, whereas more bits are assigned to high-frequency detail subbands.

3 Bit Allocation

Bit allocation among the subbands is done with an extension [1] to the GBFOS algorithm [2]. It allocates bits among the various subbands such that all allocations lie on the lower convex hull of the rate-distortion curve. The effect of Equation 4 is to bias the allocation against longer codeword indexes. As the percentage of lost bits increases, the coarse subbands that would normally have many bits assigned to them (corresponding to a deep tree) end up with a reduced number of bits, while the fine-scale subbands that would usually have few, if any, bits assigned to them, end up receiving more (but still not as many as the coarse ones, as shown in Figure 2). Essentially, in the presence of high bit loss rates, the benefits of allocating bits to low-rate detail subbands are diminished, but the benefits of high-rate coarse subbands are diminished far more rapidly.

4 Conclusion

Bit loss reduces the slope of the rate-distortion curve by a factor that is exponential in the bit rate. The exponential factor implies that short codeword indexes are affected less than long vector indexes. As losses increase, an allocation that is skewed towards low frequency subbands will shift to an allocation that is more evenly distributed. It is interesting to note that a scheme that equalizes the bit rate assigned to each subband would largely eliminate the tradeoff. For example, using higher dimension vectors for the fine-scale bands would collect the information content of even more coefficients into a single codeword.

5 Future Work

In other work, we examine the effect of bit errors on the same class of decompositions and develop methods to detect and correct those errors. In future work, we will explore methods of using channel coding to protect state-of-the-art image compression schemes, such as EZW [3] or SPIHT [4], as they are sent across lossy channels.

References

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