Brief Announcement: Ring-like DHTs and the Postage Stamp Problem

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Abstract: We explore Chord-like ring structures for Distributed Hash Tables (DHTs) and show that the Postage Stamp Problem (PSP) is equivalent to finding optimal structures for such ring topologies. We then describe a variant of the PSP that corresponds to ring-like DHTs that use greedy routing and develop an algorithm that smoothly trades off between the number of finger pointers and network diameter. We provide a dynamic programming solution to the number of nodes that can be supported as a function of the number of finger pointers and the network diameter and also note an interesting link to the Fibonacci sequence.

Categories and Subject Descriptors: C.2.4-Distributed Applications
General Terms: Algorithms, Theory.

INTRODUCTION

The Postage Stamp Problem [1] is defined as follows: given k and h, find a set of k denominations $S = \{S_1, S_2, \ldots, S_k\}$ such that:

(a) sums of h (or fewer) of these integers can realize the numbers $1, 2, 3, \ldots, N$, and

(b) the value of N in (a) is as large as possible.

The problem arises when there is space for a fixed number of stamps on an envelope and the postal service wants to issue k denominations of stamps such that the postage that can be placed on an envelope is maximized.

Ring-like DHTs can be represented as follows: let G(n; $s = \{s_1, s_2, \ldots, s_k\}$) be the network with n nodes, labeled with integers modulo n, and k links per vertex such that each node i is adjacent to k other nodes $i + s_1$, $i + s_2$, $\ldots$, $i + s_k$ (mod n). This kind of network is called a Circulant Graph. Chord [2] uses $s_i = 2^{i-1}$, where $1 \leq i \leq log_2 n$, and the resulting diameter is $log_2 n$. The set s corresponds to its finger pointers.

We now describe how Circulant Graphs are related to the Postage Stamp Problem. The k finger pointer offsets of a node correspond to the k stamp denominations. The maximum number of hops (the diameter of G) corresponds to h, the maximum number of stamps on an envelope. For a given $(k,\ h)$, S provides us with the optimal values of s_i for Chord such that the graph constructed using S maximizes the number of nodes reachable with h hops and k finger pointers.

IMPLICATIONS

Research on understanding the PSP can be applied to ring-like DHTs. For example, upper bounds to the PSP would serve as an upper bound on the maximum number of nodes for a given value of k and h:

$$N \leq \frac{(k-1)^{k-2}}{(k-2)!}\left(\frac{h}{k}\right)^k + O(h^{k-1}).$$

LARGEST DENOMINATION FIRST

Though PSP provides us with optimal set of finger pointers when using shortest path routing, ring-like DHTs follow a greedy strategy in which they minimize the identifier-space distance to the target node. This strategy leads us to define a variant of the PSP called Largest Denomination First (LDF). Given k and h find S' $=\{S'_1, S'_2, \ldots, S'_k\}$ such that:

(a) to achieve postage x, use the largest denomination $S_l \leq x$, update $x = x$ - S_l and repeat until $x = 0$,

(b) using h (or fewer) denominations can realize the numbers 1,2,. . ., N', and

(c) the value of N' in (b) is as large as possible.

RESULTS

We have developed a dynamic programming algorithm for LDF. Due to lack of space we refer the interested reader to a technical report with the details [3].

Our algorithm allows us to benefit from the tradeoff that exists between the finger pointers and the network diameter. We are able to provide tradeoff constructions using various tuples of (h, k). A network of one million nodes can be constructed using $(15, 15)$, $(12, 19)$, and $(20, 13)$.

Using our approach for a case of $h = k$ we are able to calculate the ith finger pointers as $F(2(i-1)+1)$ where $1 \leq i \leq k$ and $F(n)$ is the nth Fibonacci number. Using these as the finger pointers in place of 2^{i-1}, as in Chord, we are able to reduce the size of the finger pointers as well as the size of network diameter by 28%.

REFERENCES

[1] W.F. Lunnon, "A Postage Stamp Problem." *The Computer Journal*, pages 377–380, 1969.

[2] Ion Stoica *et al*, "Chord: A Scalable Peer-To-Peer Lookup Service for Internet Applications." *SIGCOMM 2001*, pages 149–160.

[3] Mahadev Konar and Alexander E. Mohr, "Ring-like DHTs and the Postage Stamp Problem." *Tech. Report TR-MNL-2005-01*.

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